First Year Circuit Theory Notes (MT)

Introduction and Basics

Charge: determines strength of EM force. Quantised in units of $e = 1.6\times10^{-19}$ C. Unit = Coulomb = C = As

Potential difference: energy required to move unit charge in electric field. Unit = Volt = V = J/C

$$V_{AB} = V_B - V_A = W/Q = -\int_A^B E \cdot ds.$$

Current: rate of flow of charge. $I = dQ/dt$. Unit = Ampere = A = base unit.

Power: work done per unit time. $P = \frac{dW}{dt} = \frac{d(QV)}{dt} = IV$.

Ohm's Law: For a component obeying Ohm's Law, the voltage difference across it is proportional to the current flowing through it [$V=IR$]

Ideal Voltage source: maintains a fixed voltage independent of the current it is supplying. The voltage is fixed independent of what is connected to it. A real voltage source can be thought of as an ideal source in series with a fixed resistance.

Ideal current source: supplies a fixed current independent of the voltage required to do so. The current is independent of what is connected to it. A real current source can be thought of as an ideal source in parallel to a fixed resistance.

R.M.S Values: $R.M.S(x) = \sqrt{\frac{1}{T}\int_0^T x^2\, dt}$, Conductance: $g = 1/R$ (Ω^{-1})

Conductivity: $\sigma = 1/\rho$ $(\Omega^{-1}m^{-1})$

Simple Circuits and Kirchoff's Laws

Passive Sign Convention: Sources have a + sign on the terminal the current normally leaves. Choose the direction of current you are defining as positive. For any passive component, make a + sign on the side the current is entering. Go around the loop, the sign of the voltage across a component matches the first sign you meet.

Kirchoff's current law (KCL): The sum of all currents at any node is zero (cons. charge)

Kirchoff's voltage law (KVL): Around any closed loop the sum of voltages / the net change in potential is zero.

Mesh currents: Label in 'loop' currents in all interior loops. Apply KVL around each loop (using P.S.C and KCL to get currents in shared sections). Solve for loop currents and use these to obtain everything else.

Node voltages: Choose a ground node and label all other nodes. Apply KCL to each node. (To do this write currents as $\frac{\Delta V}{R}$ where ΔV = voltage on one side of component − voltage on other side of the component). Solve for node voltages and use these to find all else.

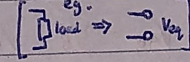
Superposition: A linear circuit with more than one current/voltage source can be analysed by considering one source at a time. The total response will be the sum of the individual responses. When analysing a source, all other voltage sources should be replaced with a wire and all other current sources, a break.

but I don't think you actually have to do

includes capacitors and inductors

Thevenin's and Norton's Theorem

Thevenin's Theorem: Any linear network of voltage/current sources and ~~resistors~~ linear elements can be written as an equivalent circuit of a perfect voltage source (V_{eq}) in series with a resistor (R_{eq}).

↳ To calculate R_{eq}, remove the load component [eg. ] and replace voltage sources with wires, and current sources with breaks. Then find the resistance of the leftover network.

↳ To calculate V_{eq}, again remove the load component. Then find the output voltage normally (may have to use mesh currents, node voltages, or superposition).

Norton's Theorem: Any linear network of voltage/current sources and resistors can be written as an equivalent circuit of a perfect current source (I_{eq}) in parallel with a resistor (R_{eq}).

↳ Find R_{eq} in the same way as in Thevenin's shown above. [$R_N = R_{TH}$]

↳ Find V_{eq} in the same way as in Thevenin's shown above. [where V_{eq} is the thevenin voltage]

↳ Find I_{eq} by $I_{eq} = \frac{V_{eq}}{R_{eq}}$ $\left[= \frac{V_{TH}}{R_N}\right]$ $\left[= \frac{V_{TH}}{R_{TH}}\right]$

Maximum Power transfer: The max power transfer from a power source (with internal resistance R_{int}) to a load (R_{load}) will be when $R_{load} = R_{int}$.

When measuring a voltage the impedance of the voltmeter should be $\gg R_{eq}$.
When measuring a current the impedance of the ammeter should be $\ll R_{eq}$

Capacitors

Capacitors store charge. The capacitance is defined by $C = \frac{Q}{V}$. Units = Farads = $\mu F - pF = A^2 s^2 J^{-1}$

Series: $\frac{1}{C_T} = \sum_{1}^{n} \frac{1}{C_n}$, parallel: $C_T = \sum_{1}^{n} C_n$. $E = \frac{1}{2} C V^2$

Inductors

Inductors oppose a change in current. They are usually coils of wire, a current passing through them results in a linked magnetic field flux ϕ. Changing the current through the circuit means changing the magnetic flux which (via Faraday's Law) results in an induced voltage: $V_{ind} = \frac{d\phi}{dt}$ which opposes the change in current. The inductance is defined via $V = L\frac{dI}{dt}$. Unit = Henrys = H = $kg\, m^2 s^{-2} A^{-2}$ $E = \frac{1}{2} L I^2$

Series: $L_T = \sum_n L_n$, Parallel: $\frac{1}{L_T} = \sum_n L_n$

Transients in RC and RL circuits

A sudden change in a RC or RL circuit, such as closing a switch, results in a time varying transient response. Apply these steps:

1. Define current directions and apply the passive sign convention as normal
2. Apply Kirchoff's Laws, remembering that $V_c = \frac{Q}{C}$ and $V_L = L\frac{dI}{dt}$
3. This leads to a set of differential eqns.

For an LCR circuit: Apply KVL → damped S.H.M with $0 = L\frac{d^2 I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C} I$ $\omega_0^2 = \frac{1}{LC}$

4. Solve these equations subject to the appropriate boundary conditions ($t=0$ and $t=\infty$) taking into account:
   - It is impossible to change the voltage on a capacitor instantly (∞ current)
   - It is impossible to change the ~~cu~~ current through an inductor instantly (∞ voltage)
5. For series RC and RL circuits the transient response is characterised by a decay/growth of current/voltage with time constants of $\tau = \frac{1}{RC}$ and $\tau = \frac{L}{R}$ respectively.

AC Theory

If we apply a time-varying current of $I = I_0 \cos(\omega t)$:

- resistor: $V_R = IR = I_0 R \cos(\omega t)$
- capacitor: $V_C = \frac{Q}{C} = \frac{1}{C}\int I\,dt = \frac{1}{\omega C} I_0 \sin(\omega t) = \frac{1}{\omega C} I_0 \cos(\omega t - \pi/2)$

  voltage lags ~~a~~ current by $\pi/2$
- Inductor: $V_L = L\frac{dI}{dt} = -\omega L I_0 \sin(\omega t) = \omega L I_0 \cos(\omega t + \pi/2)$

  Voltage leads current by $\pi/2$

important read this

We see that for capacitors and inductors, as well as defining/determining the magnitude of the voltage response, we also need the phase (how much the voltage lags/leads the current). We can encode this information into a complex impedance (Z). As Z is a complex number, it can tell us about the amplitude and phase (ϕ) of the voltage response.

Complex Impedance

$Z = Ae^{i\phi}$ or $|Z|e^{i\phi}$ ← remember this, very easy to get confused over. Also interesting: $Z = R + iX$ (resistance, reactance)

An applied current of $I = I_0\cos(\omega t)$ or $I_0 \sin(\omega t)$ can be written as $I = \mathrm{Re}(I_0 e^{i\omega t})$ or $I = \mathrm{Im}(I_0 e^{i\omega t})$ respectively. Taking the first one ($I = I_0 \cos\omega t$ - Note this can be done similarly for $I_0 \sin\omega t$ taking the Im part) we can write the generalised Ohm's Law

don't use this as it only works for $I = I_0\cos(\omega t)$. Instead use $\tilde{V} = \tilde{I}Z$ explained below. → $V = \mathrm{Re}(Z I_0 e^{i\omega t}) = |Z| I_0 \cos(\omega t + \phi)$ where Z is the complex impedance (remember Z determines both the magnitude and phase of the voltage response).

For a resistor: $Z = R$ [NB: impedances add in series and parallel the same way as resistors]

For a capacitor: $Z = \frac{1}{i\omega C}$ [as $A = \frac{1}{\omega C}$, $\phi = -\frac{\pi}{2}$ from above, $\Rightarrow Z = \frac{1}{\omega C}e^{\frac{-i\pi}{2}} = \frac{-i}{\omega C} = \frac{1}{i\omega C}$]

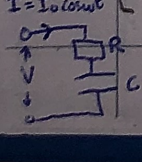
For an inductor: $Z = i\omega L$ [as $A = \omega L$, $\phi = \frac{\pi}{2}$ from above, $\Rightarrow Z = \omega L e^{i\pi/2} = \omega L i$]

symbol pronounced 'tilde'

$\tilde{V} = \tilde{I} Z$ where $\tilde{V}$ and $\tilde{I}$ are phasors (rotating vectors in the complex plane - basically just the voltage and current complexified). The actual voltages and currents are given by the projections of the phasors ($\tilde{V}$ and $\tilde{I}$) onto the real or imaginary axis (determined by how you complexified the voltage and current, eg. $I = I_0\cos(\omega t) = \mathrm{Re}(I_0 e^{i\omega t})$ or $I = I_0 \sin(\omega t) = \mathrm{Im}(I_0 e^{i\omega t})$).

Often written just as $V = IZ$ but don't forget they are phasors

eg. Find V



$Z = Z_R + Z_C = R + \frac{1}{i\omega C} = R - \frac{i}{\omega C}$

also: $Z = |Z|e^{i\phi}$

$\tilde{V} = \tilde{I}Z$

$I = I_0\cos(\omega t)$ $I = \mathrm{Re}(\tilde{I})$

$\tilde{I} = I_0 e^{i\omega t}$

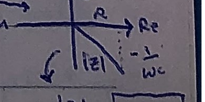
$\tilde{V} = I_0 Z e^{i\omega t} = I_0|Z| e^{i\phi} e^{i\omega t} = I_0 |Z| e^{i(\phi + \omega t)}$

$V = \mathrm{Re}(\tilde{V})$

$V = I_0|Z|\cos(\omega t + \phi)$

where

phasor diagram



$|Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

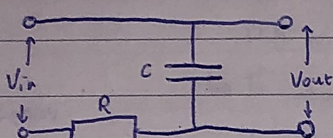
$\phi = \tan^{-1}\left(\frac{1}{\omega C R}\right)$

This is assuming input V has no impedance and output draws no current

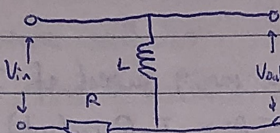
Filters: This page is messy but all the info is here

A network of passive components can be used as a filter in which an output voltage (V_{out}) across some part of the circuit is a frequency dependent fraction of some input voltage (V_{in}), such that we can write $V_{in}/V_{out} = f(\omega)$.

Low-pass RC filter

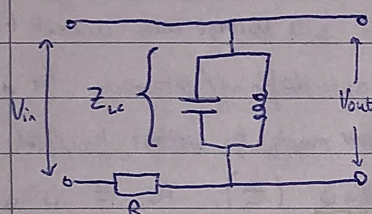


High-pass RL filter



$$\frac{\tilde{V}_{out}}{\tilde{V}_{in}} = \frac{\frac{1}{i\omega C}}{\frac{1}{i\omega C} + R} = \frac{1}{1 + i\omega RC}, \quad \frac{\tilde{V}_{out}}{\tilde{V}_{in}} = \frac{i\omega L}{i\omega L + R} = \frac{1}{1 + \frac{R}{i\omega L}}$$

Band-pass Filter



$$\frac{\tilde{V}_{out}}{\tilde{V}_{in}} = \frac{Z_{LC}}{Z_{LC} + R} = \frac{1}{1 + R/Z_{LC}}$$

$$= \frac{1}{1 + R\left(i\omega C + \frac{1}{i\omega L}\right)} \qquad \boxed{\frac{1}{Z_{LC}} = \frac{1}{i\omega L} + i\omega C}$$

$$= \frac{1}{1 + \frac{iR}{L}\left(\omega LC - \frac{1}{\omega}\right)} \longrightarrow \omega_0 LC - \frac{1}{\omega_0} = 0 \Rightarrow \boxed{\omega_0^2 = \frac{1}{LC}}$$

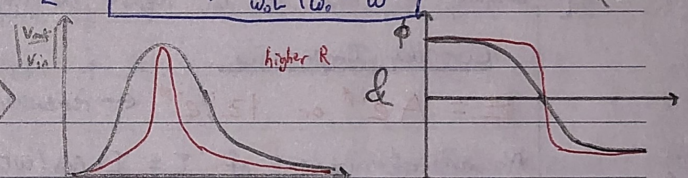
$$= \frac{1}{1 + \frac{iR}{L}\left(\frac{\omega}{\omega_0^2} - \frac{1}{\omega}\right)} \qquad Z_T = R + \frac{1}{\frac{1}{\omega L} - \omega C}$$

$$\boxed{\frac{\tilde{V}_{out}}{\tilde{V}_{in}} = \frac{1}{1 + \frac{iR}{\omega_0 L}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}} \qquad \phi = \arctan\left(\frac{R}{\frac{1}{\omega L} - \omega C}\right)$$

$\omega = \omega_0 \rightarrow \left|\frac{V_{out}}{V_{in}}\right| = 1$, phase $= 0$

$\omega \rightarrow 0 \rightarrow \left|\frac{V_{out}}{V_{in}}\right| = 0$, phase $= \pi/2$

$\omega \rightarrow \infty \rightarrow \left|\frac{V_{out}}{V_{in}}\right| = 0$, phase $= -\pi/2$



BODE PLOTS →

We can also quantify the width of the peaks of the bode plot of this band-pass filter using FWHM

Let $\omega = \omega_0 + \Delta\omega$

For FWHM $\frac{V_{out}}{V_{in}} = \frac{1}{2} = \frac{1}{1 + \frac{iR}{\omega_0 L}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$

↳ For FWHM: $\frac{R}{L\omega_0}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = 1$

$\frac{R}{L\omega_0}\left(\frac{2\Delta\omega}{\omega_0}\right) = 1$, [using $\omega = \omega_0 + \Delta\omega$ and skipping steps]

$$\boxed{\Delta\omega_{FWHM} = 2\Delta\omega = \omega_0^2 L/R}$$

Also: $Q = \frac{\omega_0}{\Delta\omega_{FWHM}} = \frac{R\omega_0}{\omega_0^2 L} = \frac{R}{L\omega_0} = R\sqrt{\frac{C}{L}}$

Power Dissipation in AC Circuits

If $I = I_0 \cos(\omega t)$ then $V = I_0 |Z| \cos(\omega t + \phi)$

$$P = IV = I_0^2 |Z| \cos(\omega t)\cos(\omega t + \phi) = I_0^2 |Z| \cos(\omega t)(\cos\omega t \cos\phi - \sin\omega t \sin\phi)$$

$$\langle P \rangle = I_0^2 |Z| (\cos\phi \langle \cos^2 \omega t \rangle - \sin\phi \langle \cos\omega t \sin\omega t \rangle)$$

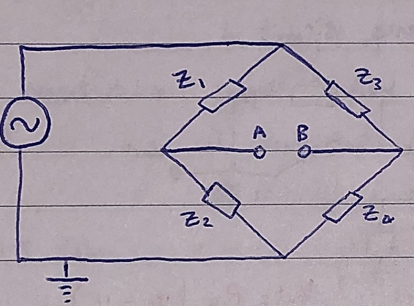
$$\langle P \rangle = I_0^2 |Z| (\cos\phi \left(\tfrac{1}{2}\right) - \sin\phi (0))$$

$$\langle P \rangle = \tfrac{1}{2} I_0^2 |Z| \cos\phi$$

$$\langle P \rangle = \tfrac{1}{2} I_0^2 |Z| \cos\phi = \tfrac{1}{2} I_0^2 \mathrm{Re}(Z) = \tfrac{1}{2} V_0 I_0 \cos\phi = V_{rms} I_{rms} \cos\phi$$

Bridge Circuits

Bridge circuits are used to determine an unknown impedance. Generally, some known impedances (eg. $Z_{1,2,3}$) are adjusted until the voltage (V_{AB}) is nulled ($=0$) (this is also known as the bridge being balanced).



$$V_A = \left(\frac{Z_2}{Z_1 + Z_2}\right) V_{in}, \quad V_B = \left(\frac{Z_4}{Z_3 + Z_4}\right) V_{in}$$

Balanced: $V_A = V_B$

$$\frac{Z_2}{Z_1 + Z_2} = \frac{Z_4}{Z_3 + Z_4}$$

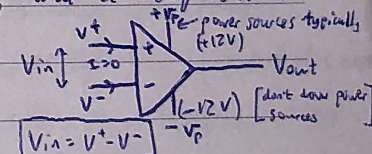
$$Z_2 Z_3 + \cancel{Z_2 Z_4} = Z_4 Z_1 + \cancel{Z_4 Z_2}$$

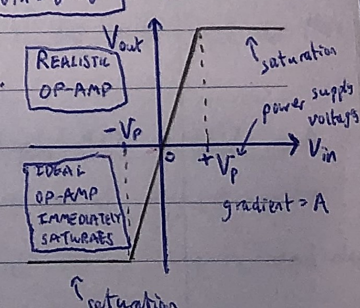
if balanced: $\boxed{Z_2 Z_3 = Z_1 Z_4}$

Operational Amplifiers (Op-amps)

An amplifier is anything that when you put some electronic signal in (voltage or current) you get out a larger version of the signal $x \rightarrow \boxed{A} \rightarrow Ax$ A is the 'gain'.

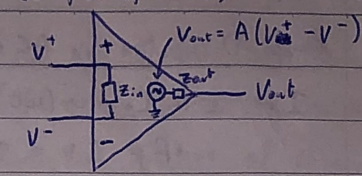
An op-amp is a type of amplifier with a high gain (typically $A = 10^5 \leftrightarrow 10^6$ but for an ideal op-amp $A = \infty$), typically used for feedback loops, and a differential input (two inputs). An op-amp is POWERED. V_{out} cannot go above its power supply voltages, when it reaches them it is known as 'saturated'. For an ideal op-amp with infinite gain, this saturation would occur immediately for any $V_{in} \neq 0$. Therefore in an ideal op-amp $V^+ = V^-$. The current in an op-amp's inputs is ZERO (it just 'senses' the voltages like a voltmeter). It may also be useful to note that an increase in V^+ leads to an increase in V_{out}, whilst an increase in V^- leads to a decrease in V_{out}. As a result, V^+ is sometimes known as the 'non-inverting input' whilst V^- is sometimes known as the 'inverting input'. Useful to look at this when you're trying to figure out circuits. Because of the way the inverting input works an op-amp has NEGATIVE FEEDBACK and they are often used to create more stable, well-controlled circuits. This can be checked by examining small fluctuations of V_{out} etc





which may be the result of a slight shift in A as a result of temp, or some other fluctuation. A small change in Vout should change V^+ and/or V^- in some way to nullify the fluctuation.

This diagram doesn't represent the actual internals but just reflects the same characteristics

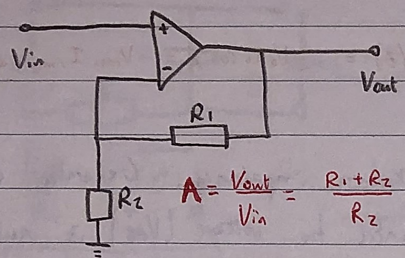


$\boxed{\text{Op-Amp Golden Rules}}$

Op-Amp Golden Rules
- ↳ 1. The inputs draw no current ($Z_{in} = \infty$)
- ↳ 2. $V_+ = V_-$ (because $A = \infty$) (in ideal)

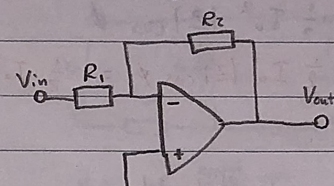
Example Circuits with OP-amps

Non-inverting amplifier



$$A = \frac{V_{out}}{V_{in}} = \frac{R_1 + R_2}{R_2}$$

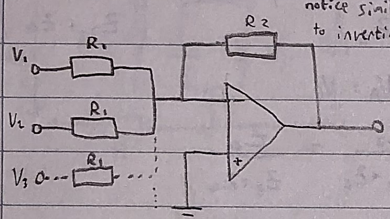
Inverting amplifier



$$A = \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$

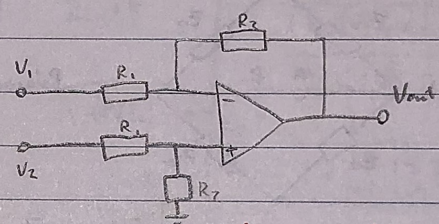
Summation circuit

notice similarity to inverting



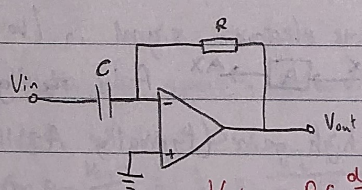
$$V_{out} = \frac{R_2}{R_1}(V_1 + V_2 + \dots)$$

Difference circuit



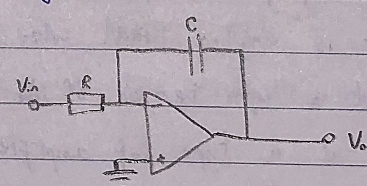
$$V_{out} = \frac{R_2}{R_1}(V_2 - V_1)$$

Differentiation circuit



$$V_{out} = -RC\frac{dV_{in}}{dt}$$

Integration circuit



$$V_{out} = -\frac{1}{RC}\int_0^t V_{in}\,dt$$

$$V_{out} = \frac{1}{C}\int I\,dt$$
$$= \frac{1}{C}\int \frac{V_{in}}{R}\,dt$$
$$=$$